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Visio, perspectiva and representation of space in the sixteenth-century linear perspective treatises

SAITO, FUMIKAZU

History of Science/CESIMA/PUCSP

Mathematics Education/History of Mathematics-HEEMa/PUCSP

fsaito@pucsp.br

Introduction

Currently discussion concerning the representation and visualization of objects in space are relatively settled. In this work, I use the terms "visualization" and "representation" in their very restricted meaning. By visualization I mean the act or the effect of seeing with the eyes which can be explained by different theories of vision. And by representation as the act of reproducing what is seen, or visualized, in the pictorial form according to a set of rules associated with a theory of pictorial representation such as the linear perspective.

Thus, broadly speaking, we usually accept without question that the space or object represented on a flat surface corresponds to the space and the object of our sensory experience. The round table which is represented in its elliptical shape in a painting, for example, does not cause any strangeness. We usually associate that elliptical form on the painting with the circular shape of a round table, and then we establish a correspondence between what is seen and what is represented. In technical terms, we can say that the circular table top is seen elliptical in our visual space. Then this shape is transported to the plane of a frame. And finally, the round table is displayed or represented in an elliptical shape.

However, in this example, how can we assure that the table top represented in the frame corresponds, in fact, to a circular table top of our sensory experience? Only the painter could assert it, and it is up to the viewer to accept (or not) that such a representation corresponds, in fact, to the represented object.

This ambiguity is essentially epistemological and points to a broader conceptual change in the very notion of physical and geometrical spaces from sixteenth century onward.[1] In this

[1] On this, see: Panofsky, *A perspectiva como forma simbólica*; see also: Saito, "O espaço nas origens da ciência moderna e a sua representação geométrica segundo a *perspectiva naturalis* e *artificialis*", *in* P. P. Silva & B. G.

respect, it is worth remembering that, historically, the disputes concerning the theories dealing with the relation between vision and the objects of visual perception along with their representation were not simple and obvious.[2] However, in the sixteenth century this issue was discussed on the basis of a new developments in the studies on nature and arts (*technai*). Optics and its new developments concerning linear perspective, which introduced new standards of "accurate" drawing and new criteria to "optical truth", enlarged and redefined the space of visibility. A set of documents dealing with the codification of the pictorial technique in geometrical terms provides evidences that linear perspective intensified the debate on natural and artificial vision.[3]

Between natural and artificial vision

A rich literature consisting on the study on *perspectiva* in general published in mid-16th century reveals that painters and scholars such as Sebastiano Serlio (1475-1554), Pietro di Giacomo Cataneo (1510-1574), Daniele Barbaro (1514-1570), Egnatio Danti (1536-1586), among many others, were familiar with some of the classical doctrines regarding the visual process. Such authors shared different ideas about the apprehension and the perception of natural reality and established a rich dialogue with scholars dedicated to optics research.

Part of the interest in these theoretical considerations was related to the place that the linear perspective occupied in the organization of knowledge. Optics and its associated field, geometry, provided the linear perspective with the necessary fundamentals for it to be considered a liberal art.[4] In addition, because the linear perspective used geometrical shapes to

Figueiredo (orgs.). *Anais eletrônicos do 14 Seminário Nacional de História da Ciência e da Tecnologia*, 1-13; Saito, "Alguns aspectos da noção de espaço geométrico no século XVI a partir de um estudo preliminar de duas obras de Francesco Patrizi da Cherso", *in XI Seminário Nacional de História da Matemática*, 1-19.

[2] See: Aiken, "Truth images: from the Technical Drawings of Ibn Al-Razzaz Al-Jazari, Campanus of Novara, and Giovanni De'Dondi to the Perspective Projection of Leon Battista Alberti", *Viator*, 25, 325-359; Hamou, *La vision perspective (1435-1740): L'art et la science du regard, de la Renaissance à l'âge classique*; Simon, *Le regard, l'être et l'apparence dans l'optique de l'antiquité*; Simon, *Archéologie de la vision: l'optique, le corps, la peinture*; Summers, *The Judgement of Sense: Renaissance Naturalism and the Rise of Aesthetics*; Wade, *Perception and illusion: historical perspectives*.

[3] Baxandall,*Painting and Experience in Fifteenth-Century Italy*; Edgerton Jr, "The Renaissance Development of the Scientific Illustration", *in* J. W. Shirley & F. D. Hoeniger (eds.), *Science and Arts in the Renaissance*, 168-197; See also: Saito, *O telescópio na magia natural de Giambattista della Porta*, 137-172; Saito, "Arte, ciência e magia: manipulando o espaço no século XVI", *in* M. M. Moraes (org.), *Formas Imagens Sons: O Universo Cultural da Obra de Arte*, 222-231.

[4] On Optics, Astronomy and Music as mixed sciences, see: Gagné, "Du *Quadrivium* aux *Scientaie Mediae*", *in* Insitute d'étude medieval, *Arts Liberaux et Philosophie au Moyen Age. Actes du IVe Congrès International de Philosophie Médiévale*, 975-86; Nascimento, *De Tomás de Aquino a Galileu*, 13-87; Vescovini, "L'inserimento della 'perspectiva' tra le arti del quadrivio", *in* Insitute d'étude medieval, *Arts Liberaux et Philosophie au Moyen Age. op.cit.*, 969-74.

represent the physiological visual space, it was thought of as one of the many ramifications of optics.[5]

Note that the term *perspectiva* was the Latin translation of the Greek term *optiké*, meaning direct or distinct vision. Medieval treatises adopted the designation *perspectiva* alongside other terms, such as *aspectibus* and *visu*, and it coexisted with the term that named the pictorial technique.[6] To distinguish them, "common" or "natural" perspective was commonly opposed to the *perspectiva artificialis* of painters.

The terms aspectibus, visu, optica, and perspectiva were used to refer to the study of optics in general. [7] During the Middle Ages, the term perspectiva was employed to refer to a set of theories of vision. Aspectibus, in turn, was usually used to refer to the problems of visual appearance. Futhermore, it related to the optical studies that were developed by Arabic tradition (9th to 12th centuries), especially to the theory developed by Alhazen (965-1040). Visu was usually closely associated to the Greek word optike and was employed to refer to the sight and its properties regarding the problems of sight and the evidences of seeing sensible things. And the vernacular translation "perspective" was used to appoint the geometrical study the pictorial technique by which one could create the illusion of depth on a two-dimensional plane.[8]

The analysis of different approaches related to the linear perspective reveals that it is impossible to establish a clear boundary between *perspectiva artificialis* and *perspectiva naturalis* because both are related in various ways in a large spectrum of possibilities. However, this same analysis shows signs of a gradual separation between these two expressions of optics in the mid-17th century. Although some aspects of the vision still continued to be part of treatises on linear perspective, others connected to the physiology and anatomy of the eye began to be cast aside.[9]

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[5] See: Saito, "O espaço nas origens da ciência moderna...".
[6] Hamou, "Introduction", *in La vision perspective. op.cit.*, 7; see also Simon, *Le regard. op.cit.*, 73.
[7] Lindberg, *A Catalogue of Medieval and Renaissance Optica Manuscripts*.
[8] Vescovini, "Vision et réalité dans la perspective au XIVe siècle", *Micrologus: Natura, scienze e società*, V, 161-180; on *prospetto* e *aspetto*, see: Frangenberg, "The Image and the Moving Eye: Jean Pélerin (Viator) to Guidobaldo del Monte", *Journal of the Warburg and Courtauld Institutes*, 49, 150-171.
[9] See, for example: Alberti, *Da pintura*; Barbaro, *La pratica della perspettiva di monsignor Daniel Barbaro ...*; Caus, *La perspective avec la raison des ombres et miroirs ...*; Cigoli, *Prospettiva pratica*, *in* Camerota, (ed.), *Linear Perspective in the Age of Galileo: Ludovico Cigoli's* Prospettiva pratica; Cousin, *Livre de perspective de Jehan Cousin, senenois, maistre painctre à Paris*; Del Monte, *Guidubaldi è Marchionibus Montis Perspectivae libri sex*; Dubreuil, *Perspective practical or a plain and easie method of true and lively representing all things to the eye at a distance, by the exact rules of art ...*; Dürer, A. *Instituciones de Geometría*; Moxon, *Practical perspective, or , Perspective made easie teaching by the opticks...*; Nicerón, *La perspective curieuse ....*; Pélerin (Viator), *De artificiali perspective*; Vedreman frisius, *La tres-noble perspective.....*

Treatises such as *La pratica della perspettiva* by Daniele Barbaro bring several elements found in traditional optics treatises. In *La pratica della perspettiva*, there are references to visual rays and other properties of vision. However, some considerations on the anatomical and physiological study of the eye, are eliminated. On the other hand, the *Livre de perspective* (1560) by Jean Cousin (*c.*1490-*c.*1560) only addresses geometric tracings and does not mention any other aspects related to vision. By contrast, the treatise *La perspective* (1611) by Salomon de Caus (1576-1626) includes some considerations about the anatomy of the eye (particularly with regard to its form) linked to the range of vision. For example, according to Caus, depending on the depth of the optic nerve in relation to the eye's surface, the object seen could be larger or smaller. Thus, each person would perceive one same object with different sizes, which would be one of the reasons why some people would need to wear glasses.

However, other treatises dedicated more exclusively to optical issues slowly began to give greater importance to light, with the linear perspective beginning to consider other issues related to shadow tracing. In addition, questions on the anatomy and physiology of the eye once found in this literature gradually migrated to other treatises related to medicine or physics.[10] Treatises such as that by Johannes Kepler (1571-1630) began to disregard these issues, noting that it was not the job of the optics scholar to explain how images, projected on the retina of the eye, entered by the optic nerve and traveled to the brain. [11] Other natural philosophers, such as Isaac Newton (1643-1727), started to pay attention only to the physical and geometrical aspects of optics, reducing it gradually to the study of light.[12]

Regarding the treatises on *perspectiva artificialis*, we note that over the 16th century, they began to disregard the materiality of visual rays and light beams, treating the linear perspective in its essentially geometric aspects. This movement unfolded into two branches of research, one aimed at the search for a "general rule" for perspective drawing and the other aimed at exploring the geometric properties of tracings. In relation to the first branch, scholars such as Danti and Giacomo Vignola (1507-1573) dedicated themselves to finding a rule for the

[10] See, for example: Aguilonius, *Francisci Aguilonii e Societate Iesu opticorum libri sex. Philosophis iuxtà ac Mathematicis utiles*; Euclid, *Evclidis Optica & Catoptrica...*; Fabricius de Acquapendente, *De visione voce auditu*; Gregory, *Optica promota, seu Abdita radiorum reflexorum &c...*; Pecham, *John Pecham and the Science of Optics: Perspectiva communis*; Powel, *Elementa opticae nova, facili, 7c; compediosâ methodo explicata...*; Risner (ed.), *Opticae thesaurus...*; Scheiner, *Oculus, hoc est...*

[11] Kepler, *Les fondements de l'optique moderne: Paralipomènes a Vitellion*, 317-318; Field, "Two Mathematical Inventions in Kepler's *Ad Vitellionem paralipomena*", *Studies in History and Philosophy of Science*, 17, 4, 449-468.

[12] Newton, *Óptica.*

costruzione legittima.[13] With respect to the second branch, scholars such as Guidobaldo del Monte (1545-1607) sought an essentially geometric treatment of *perspectiva*.[14] Subsequently, other studies related to anamorphosis[15] and the different ways in which it was possible to distort an image began to compose the list of subjects of geometricians, leading them to discuss and explore its essential geometric properties.[16] Along with other procedures connected to astronomy, cartography, and geography, the path was opened to explore geometric projections, inaugurating yet another chapter of geometry.

On that path, in which new research fields were outlined, the issue of the ontological status of artificial vision was implicit. [17] The scholars at that the time criticized the artificiality resulting from the geometrization of visual space and pointed to four restrictive reasons. According to them, the technique of perspective: 1) limited the visual field; 2) immobilized the eye; 3) ignored the sphericity of the visual field; and 4) reduced eyesight to monocular vision. These four reasons stressed the discrepancy between natural and artificial vision, leading many scholars of *perspectiva artificialis* to formulate a freer concept of perspective that admitted methods of optical adjustment. [18] In fact, evidence in this regard is found in different treatises published in that period, notably in *La pratica della perspettiva* by Barbaro.

In Chapter I, Book I of *La pratica della perspettiva*, Barbaro contends that when addressing perspective, "... we must consider not only what the eye sees but also how it sees..."[19], thus establishing a close relationship between vision and pictorial representation. This relationship is emphasized in the second chapter of the work, which addresses, albeit briefly, the functioning of the eye. It distinguishes between two types of vision, called 1) "simply looking and seeing"; and 2) "careful and accurate". [20] According to Barbaro, the first type refers to the operation of nature and the second type, to its reason. It is very likely that in establishing

[13] Antonini, (ed.), *Jacopo Barozzi da Vignola: La regola dei cinque ordini, Le due regole della prospettiva pratica nella edizione del 1828 proposta da Carlo Antonini*; Frangenberg, "Egnatio Danti's Optics. Cinquecento Aristotelism and the Medieval Tradition", *Nuncius*, III, 1, 3-38; Frangenberg, "Perspective Aristotelianism: three case-studies of Cinquecento visual theory", *Journal of the Warburg and Courtauld Institutes*, 54, 150-158.
[14] Del Monte, *Perspectivae libri sex*, *op.cit.*
[15] See, for example: Schott, *Magia universalis, naturae et artis....*
[16] Nicéron. *La pespective curieuse, op.cit.*
[17] On this, see: Saito, "O espaço nas origens da ciência moderna..."; Saito, "Alguns aspectos da noção de espaço geométrico no século XVI...". See also: Edgerton, *The heritage of Giotto's geometry: art and science on the eve of the scientific revolution*; Edgerton, *The Mirror, the Window, and the Telescope: How Renaissance Linear Perspective Changed Our Vision of the Universe*; Floriênski, *A perspectiva inversa*.
[18] See: Frangenberg, "The Angle of Vision: problems of perspectival representation in the fifteenth and sixteenth centuries", *Renaissance Studies*, 6, 1, 1-45; Frangenberg, "Optical correction in sixteenth-century theory and practice", *Renaissance Studies*, 7, 2, 205-228; Veltman. "Panofsky's Perspective: A Half Century Later", *in* M. D. Emiliani (ed.), *Atti del convegno internazionale di studi: la prospettiva rinascimentale, Milan 1977* , 565-584.
[19] Barbaro, *La pratica della perspettiva.op.cit.*, 5.
[20] Ibid., 6.

this distinction, he was referring to the geometrization of *visio* because he discarded physiological considerations of "the natural way of seeing". He did so when he claimed that "it is not for us to discuss whether vision occurs through reception or transmission"[21] because vision occurs by way of a visual pyramid, in which the vertex is the center of the eye (point) and the observed object is the base (visible surface).[22]

When Barbaro affirmed that it was irrelevant whether the visual process occurred through the emission of visual rays or the reception of light beams, he avoided discussing the physiological and anatomical aspects of the eye in the visual process. This economy in the explanation of the refraction of visual lines in the eye can be certainly understood through the influence of Euclid's *Optics*. However, Barbaro was aware that the visual phenomenon was not essentially geometric, given that he claims that "one cannot see from all angles" [23], pointing to the difference between natural and mathematical angles. Here, Barbaro is referring to the traditional Aristotelian distinction between mathematical and natural (material) objects. Thus, he concludes that perspective considers the reason of signs, lines, and natural angles. [24]

Barbaro's deliberations are evidence that the linear perspective was not an essentially geometric domain but, rather, "mixed mathematics" because he considered the visual ray, or light beams, in two ways: naturally and geometrically. The visual line (natural) was called the "visual ray" or the "light beam" and was deprived of width (because they were geometric lines). However, to the extent that it was through the visual line that things were seen, it should also be considered natural.[25]

These considerations lead the reader to accept that the visual ray and light beams can be treated geometrically without, however, disregarding their sensitive and physical effect in the visual process. Therefore, the linear perspective has tended to harmonize natural and artificial aspects linked to vision. Nonetheless, by disregarding the physical and physiological treatment of vision, favoring only its geometric aspects, the linear perspective widened the gap between natural and artificial vision. This can be verified in at least two points in Barbaro's treatise: 1) the eye is reduced to a point; and 2) the visual cone is replaced by a visual pyramid, thus modifying the visual field (Figure 1).

[21] Ibid., 6.
[22] Ibid., 6.
[23] Ibid., 7.
[24] Ibid., 8.
[25] Ibid., 8.

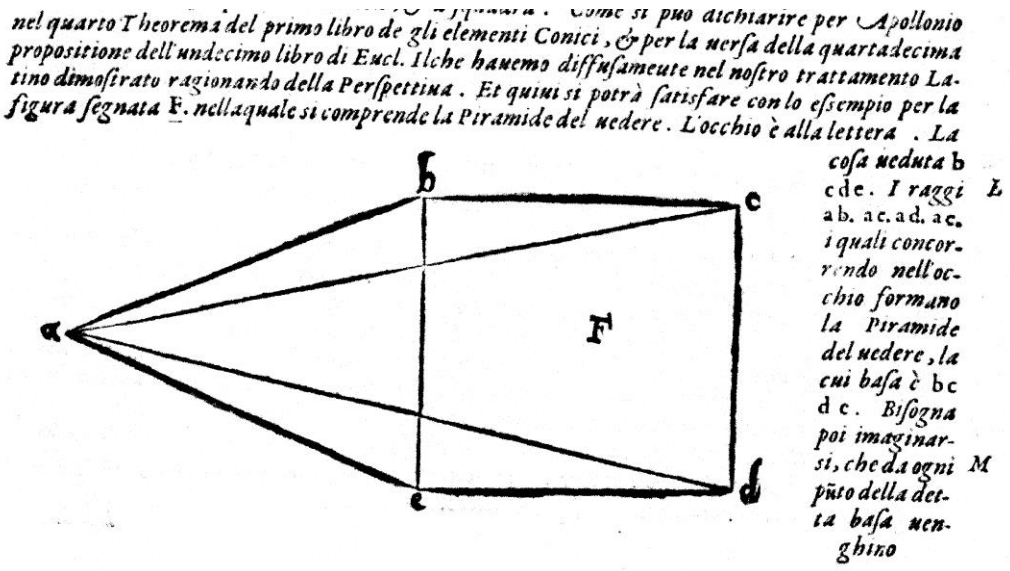

Figure 1: Visual Piramid [26]

By reducing the eye to a point, Barbaro casts aside considerations of natural vision. He reduces the visual process to artificial, monocular vision, with a very restricted visual field. [27] Thus, all attributes associated with the visible object are "frozen" and calculated from a unique point of view. [28] Because he replaces the cone (circular base) with a visual pyramid (square base), Barbaro changes the circular visual field to a quadrangular field. This change indicates that *perspectiva artificialis* had begun to diverge from *perspectiva naturalis*, establishing a split between theories of vision and representation.

Final remarks

La pratica della perspettiva, among many other treatises on perspective published in the sixteenth century, points to the split between natural and artificial vision. However, this split is not complete. The procedure adopted by Barbaro, refracting the new conceptions of nature that began to gain importance in his time, reflects ancient and traditional knowledge linked to the nature of vision. One can say that these two expressions of perspective are harmonized in *La pratica della perspettiva* by Barbaro which sometimes approach and sometimes move away from ancient conceptions of optics.

[26] Ibid., 8
[27] See: Field, "Piero della Francesca's treatment of edge distortion", *Journal of the Warbourg and Courtauld Institutes*, 49, 69-99; Field, *The Invention of Infinity: Mathematics and Art in the Renaissance*; Raynauld, "Les débats sur les fondements de la perspective linéaire de Piero della Francesca à Egnatio Danti: un cas de mathématisation à rebours", *Early Science and Medicine*, 15, 474-504.
[28] Barbaro, *La pratica della perspettiva.op.cit.*, 8.

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